



Fig. 1 Equilibrium of the node.

rors); i.e., there are no stress resultant discontinuities at the nodes. This is true for the structural system of the example or for any other system.

Parenthetically, it is worthwhile to mention that the stiffness matrix and the generalized forces of the structural elements of the example are exact since the third-order polynomial is the exact deflection pattern for a straight flexural beam element with constant ( $EI$ ). This is not the case when approximate deflection patterns are used as, for example, in the work on shells of revolution by the Massachusetts Institute of Technology group.

Consider the node joining elements 1 and 2 of the example (Fig. 1) with the stress resultants taken from columns 1 and 2 of the last equation of the Note. The discontinuities in the bending moments ( $0.009pl^2$ ) and in shear forces ( $0.014pl$ ) are exclusively due to computational errors. Theoretically there should not be such discontinuities since this situation corresponds precisely to the satisfaction of the node equilibrium conditions. The writer worked out this example by a standard matrix structural analysis FORTRAN program. It was found that the stress resultants, from each side of the nodes, agreed with the exact solution with five significant figures.

In conclusion, contrary to the assertion of the Note that "although this example is too simple to draw general conclusions from . . .," it can be conclusively asserted that the method is an exact one even for shells of revolution. For this type of structural system the only approximation is due to the use of approximate deformation functions for the shell elements. However this does not cause any stress resultant discontinuities at the nodes.

#### References

- <sup>1</sup> Stricklin, J. A., "Computation of Stress Resultants from the Element Stiffness Matrices," *AIAA Journal*, Vol. 4, No. 6, June 1966, pp. 1095-1096.
- <sup>2</sup> Filho, F. V., "Matrix Analysis of Plane Rigid Frames," *Transactions of the American Society of Civil Engineers*, Vol. 126-II, 1961, pp. 214-227.
- <sup>3</sup> Filho, F. V., "Discussion on Generalized Displacements in Structural Analysis, by J. L. Meek," *Journal of the Structural Division, Proceedings of the American Society of Civil Engineers*, Vol. 88, ST6, 1963, pp. 303-316.

## Reply to F. V. Filho

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PROFESSOR Filho has studied my technical note in detail and retained more significant figures in his calculations. Although the equations used in my technical note<sup>1</sup> and Ref. 2 of Professor Filho's comments are the same, our objectives were quite different. In particular my objective was to obtain

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the stress resultants after computation of the generalized displacements. My assertion about not being able to draw general conclusions from a simple example has been verified by more recent studies on shells of revolution.<sup>2</sup> It was found that my method yields good results when the geometry of the shell is accurately represented but yields poor results when the shell is represented by conical segments. Based on this more recent data we can say that the method is theoretically correct but sensitive to geometric approximation.

The statement "since the third order polynomial is the exact deflection pattern for a straight flexural beam element with constant ( $EI$ )," is not correct. For example, the exact deflection pattern for a uniform beam under a uniform loading is a fourth-order polynomial. In summary, we used the same equations to reach different objectives. Otherwise, I believe the limitation as stated in Ref. 1 should remain.

#### References

- <sup>1</sup> Stricklin, J. A., "Computation of Stress Resultants from the Element Stiffness Matrices," *AIAA Journal*, Vol. 4, No. 6, June 1966, pp. 1095-1096.
- <sup>2</sup> Navaratna, D. R., "Computation of Stress Resultants in Finite Element Analysis," *AIAA Journal*, Vol. 4, No. 11, Nov. 1966, pp. 2058-2060.

## Torque on a Satellite in a General Gravitational Field

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SCHLEGEL<sup>1</sup> has properly pointed out that I was careless (he was kind enough not to use the word) in an earlier derivation of the gravitational torque on a satellite in a non-inverse square field.<sup>†</sup> He provides a correct development and, in particular, gives forms for the body components of torque in the neighborhood of the oblate earth. His results are expressed partly in terms of body orientation parametrized by Euler angles, though portions of the equations are simplified considerably by his leaving them in terms of the direction cosines, which he denotes by  $m_{\alpha\beta}$  ( $\alpha, \beta = 1, 2, 3$ ).

In this Note, I would like to give an alternative derivation which is somewhat more compact and is more straightforward, in my opinion, in regard to the development of the torque components from the potential. It leads to a compact matrix form for the result that is more general in several particulars; a form that also is extremely convenient for the numerical computation of these torques for the purposes of digital simulation.

Denote by  $U(\mathbf{R})$  the specific potential at the center of mass of a material system due to the gravitational field in which it is immersed,  $\mathbf{R}$  locating this center of mass with respect to the center of the earth or other central body. Denote by  $U(\mathbf{R} + \boldsymbol{\rho})$  the specific potential at a nearby point. Integrating the latter over the material system and using a well-known expansion from vector analysis, valid when the gradient operator is expressed in rectangular coordinates, the spe-

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<sup>†</sup> Several others have pointed out to me the fallacy of my original development, and I wish to take this opportunity to express to them my genuine appreciation.